

Modulated replica symmetry breaking schemes for antiferrimagnetic spin glasses

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Abstract. We define modulated replica symmetry breaking (*RSB*) schemes which combine tree- and wave-like structures. A modulated scheme and unmodulated *RSB* are evaluated at 1-step level for a semiconductor model with antiferromagnetic Korenblit-Shender interaction. By comparison of the free energies we find evidence that a $T = 0$ phase transition in the ferrimagnetic phase leads to a transition between the different *RSB*-schemes. An embedding factor of Parisi block matrices with sublattice-asymmetrical size is employed as a new variational parameter in the modulated scheme.

PACS. 68.35.Rh Phase transitions and critical phenomena – 75.10.Nr Spin glass and other random models

1 Introduction

Hierarchical tree-structures and replica symmetry breaking (*RSB*) [1,2] are celebrated features in the theory of magnetic systems with random and frustrated interactions of infinite range. The role of these symmetry breakings for short-range spin glasses was hotly debated over recent years in the work of different groups [3,4].

A puzzling question over many years concerned the existence of variants or alternatives for the Parisi *RSB*-scheme. The latter proved to be very robust however. As a consequence it appeared to be a difficult task to find relevant variables which perturb or change the scheme and in addition lead to new selfconsistent solutions.

In this article we define and apply *RSB*-schemes which show a kind of wave-like modulation in addition to the tree-structure [1,2], the modulation being controlled by an additional parameter. We work in the context of a two-sublattice model with infinite-range interactions only between spins belonging to different sublattices A or B. We show that a sufficiently strong asymmetry between A- and B-type sublattices, as for example realized by ferrimagnetic order, turns out to be the origin of these new self-consistent solutions with modulated *RSB*.

Sublattice models with infinite-range frustrated interactions, and allowing spin glass phases for example, were first defined and theoretically analyzed in replica-symmetric (*RS*)-approximation by Korenblit and Shender [5] (*KS*-model). In subsequent papers Fedorov, Korenblit, and Shender also evaluated the infinite-*RSB* theory for a temperature range close to and below T_c .

These authors described the relevant structure of the replicated field theory, determined phase diagrams in external fields, and discussed applications, for example to strongly disordered frustrated antiferromagnets $\text{Fe}_x\text{Mg}_{1-x}\text{Cl}_2$ [6–8].

The *KS*-model successfully offers the description of transitions between spin glass and antiferromagnetic order (or glassy ferrimagnetic order in a field) in spite of an infinite-range interaction in real space. Spatially staggered order is allowed by letting the interaction work only between different sublattices. The field of application includes two-component magnets as well as standard antiferromagnets, where the staggered magnetic order itself defines sublattices. Moreover, range-free interactions cannot distinguish spatial positions and consequently unite the mean field picture of sublattice systems with another class of systems having just an equal number of mutually interacting but arbitrarily placed A- and B-spins.

The *KS*-model effectively mirrors the phase diagram of the Sherrington-Kirkpatrick (*SK*-)model [9] to the antiferromagnetic side, still allowing to retrieve ferromagnetic solutions.

Additional intra-lattice interactions can also be dealt with in refined models. Fyodorov, Korenblit, and Shender [10] applied such generalized models to $\text{Fe}_x\text{Mn}_{1-x}\text{TiO}_3$ alloys with purely antiferromagnetic inter-layer but frustrated intra-layer interactions generated by alloying ferromagnetic FeTiO_3 and antiferromagnetic MnTiO_3 -layers.

In this paper we focus on phase transitions from spin glass to ferrimagnetic order, driven by inter-layer antiferromagnetic interactions. These transitions are frequent

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physical phenomena and concern a wide range of different microscopic models. Even in cases when quenched disorder is weak or absent, spin glass models can have the power to mimic behaviour of clean but geometrically frustrated systems [11–13].

As an application of our asymmetric *RSB*-scheme we employ (in section 4) a generalization of a model designed for a CdTe/Cd_{1-x}Mn_xTe layered system [14]. In this model magnetic exciton polarons near the interface provide a confined magnetic field in the spin glass material Cd_{1-x}Mn_xTe. The previous Villain Ising pseudospin model has been used to explain experimental features for the pure spin glass phase of $x = 0.33$ samples [14]. The generalized form can be used to study as well the competition between glassy antiferromagnetic and pure spin glass phases; the instability driven by the antiferromagnetic interaction is known to occur at much higher manganese concentrations. Beyond this transition and in a field, which breaks the A-B symmetry, our modulated *RSB*-scheme (*SES*) happens to become the favourable solution (as far as the 1-step symmetry breaking can tell).

Beyond the present application to antiferromagnetic instabilities and ferrimagnetic glassy phases, our results suggest that the modulated *RSB*-schemes could also serve as a basis to describe excited states in finite-range spin glasses. This suggestion refers to incorporate not only simple spatially staggered variations as a characterizing feature of the present modulated *RSB*-scheme, but perhaps also combine more general spatial and time-dependent fluctuations with replica symmetry breaking schemes.

2 The two-sublattice spin glass model with competing antiferromagnetic- and ferrimagnetic order

A class of Hamiltonians, for which the modulated *RSB*-scheme is constructed, is given by the *KS*-model in an external field $h(r)$

$$\mathcal{H} = - \sum_{i_A=1}^N \sum_{j_B=1}^N J_{i_A, j_B} S(r_{i_A}) S(r_{j_B}) + \sum_{i_A} h(r_{i_A}) S(r_{i_A}) + \sum_{i_B} h(r_{i_B}) S(r_{i_B}), \quad (1)$$

where the partially frustrated random interaction J_{i_A, j_B} is chosen to obey a Gaussian distribution $P(J_{i_A, j_B}) = \exp(-N(J_{i_A, j_B} + J_{af}/N)^2 / (2J^2)) \sqrt{N/(2\pi J)}$. Centered at a negative mean coupling $\langle J_{i_A, j_B} \rangle = -J_{af} < 0$, the sublattice-interaction permits glassy antiferromagnetic order. Glassy ferrimagnetic order with lifted $A \leftrightarrow B$ symmetry results for example when a homogeneous field is applied or when spins of different lengths (different spin quantum numbers in quantum models) happen to be located on different sublattices. Modulated *RSB* should also be considered for model classes including interactions of different types of localized spins, for example tight-binding electron spins coupled to ionic spins S ; initially mobile

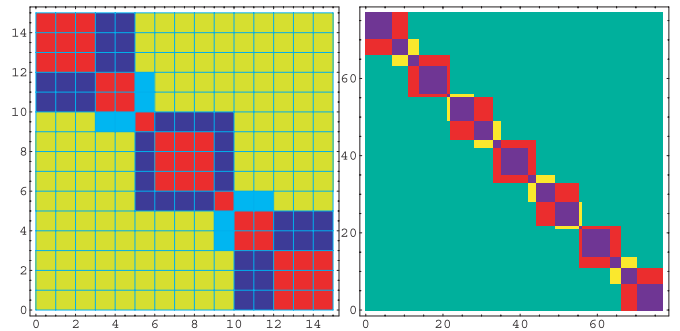


Fig. 1. Simple examples of modulated saddle point matrices $Q_{AB} = Q_A + Q_B$ with $m_A = 3$, $m_B = 5$, and $n = 15$ (left figure) and $m_A = 7$, $m_B = 11$, and $n = 77$ (right figure) in one-step *RSB*. Four different regions are shown with entries $q_{1A} + q_{1B}$, generated by the overlap-areas of $m_A \times m_A$ - and $m_B \times m_B$ -sized block matrices along the diagonal, with $q_{1A} + q_{2B}$ and $q_{2A} + q_{1B}$ generated by their nonoverlap-regions, and $q_{2A} + q_{2B}$ belonging to the superposition of off-diagonal A - and B -elements.

carriers, which localize due to their interaction with ionic spins, may not be able to fit the *RSB* glassy order. This example reaches far beyond the classical model (1). In the present work we focus exclusively on model (1) with minimal inequivalence of sites (such as being of A - and B -type), which requires a hybrid modulated form of *RSB* in a solvable classical model and hence reveals a coupling of replica- and real space.

In replica theory [15], which we use here, all spin variables acquire a replica-index a , $S \rightarrow S^a$. After elimination of the microscopic spins the corresponding effective Lagrangian of the replica theory [9] is given in terms of Hubbard-Stratonovich fields [5]. The *SK*-model interaction requires one such field [9], $\tilde{Q}^{a,b}$. Its statistical average $Q^{ab} \equiv \langle \tilde{Q}^{ab} \rangle = \langle S_i^a S_i^b \rangle$ describes glassy order [1,2] in addition to a homogeneous magnetization $M = \langle S_i^a \rangle$, which can be finite in case of partial frustration. The *KS*-model however involves for each sublattice $\kappa = A, B$ a magnetization $M_\kappa \equiv \langle S_{i_\kappa}^a \rangle$ and $Q_\kappa^{ab} \equiv \langle \tilde{Q}_\kappa^{a,b} \rangle$, and a field $\tilde{Q}_3^{a,b}$ which couples the sublattices [5]. The averaged matrix $Q_{AB} \equiv -i Q_3 \equiv -i \langle \tilde{Q}_3 \rangle$ turns out to be equal to $Q_A + Q_B$, where Q_A and Q_B inevitably show sublattice-splitting of their entries $q_A \neq q_B$ in *ferrimagnetic* phases, together with $|M_A| \neq |M_B|$. The *size* of their block-diagonal matrices, characterized in *RSB* by a Parisi parameter m [1], may also develop a sublattice-asymmetry. Thus, at 1*RSB*-level two order parameters for each sublattice A or B , hence $\{q_{1A}, q_{2A}, q_{1B}, q_{2B}\}$, denoting matrix elements of Q_A and Q_B , and two Parisi-parameters m_A, m_B need to be considered.

A simple illustration for the matrix $Q_{AB} = -i Q_3$ is displayed in Figure 1. Note that for simplicity it is not shown that elements on the diagonal vanish (while, for example in fermionic spin glasses, these elements equal 1 at half-filling; this detail can trivially be accounted for in the trace formulas below, but is of no relevance for our present application).

Unmodulated replica-symmetry breaking scheme: In the unmodulated 1RSB scheme one chooses $m_A = m_B$.

3 Modulated replica symmetry breaking schemes

We consider the superposition of two diagonal Parisi-block matrices with sizes $m_A \times m_A$ and $m_B \times m_B$ such that a rational embedding factor γ denotes the number of smaller blocks (let $m_A < m_B$) *fully embedded* inside larger m_B -boxes, i.e. without being intersected by the latter ones, divided (normalized) by the number of larger boxes inside the entire ($n \times n$) host matrix (the example of Fig. 1 shows $\gamma = 1$ (left figure) and $\gamma = \frac{5}{7}$ (right figure)).

We distinguish now a single embedding scheme, for which the host size n is restricted to be the least common multiple of m_A and m_B , and multiple embedding schemes, which align k -times such structures along the diagonal.

For all of $n, m_A, m_B, n/m_A$, and n/m_B integral and $m_A < m_B$, the embedding parameter γ is given by

$$\gamma = (n/m_A - (n/m_B - 1))/(n/m_B). \quad (2)$$

Let us now turn to the important replica limit $n \rightarrow 0$.

3.1 Single-embedding modulated scheme (SES)

When the limit $n \rightarrow 0$ is approached, as required in replica theory, equation (2) and the integral constraints associated with it must be relaxed. This allows to obtain a finite nontrivial free energy. To this end we choose γ as a free variational parameter like m_A and m_B . To obtain the free energy F for this modulated *RSB*-scheme we need the trace of the square of such super-imposed Parisi matrices (see Eq. (6) below). The free energy involves the limit as n goes to zero of $(1/n)$ times a sum of traces over the Q^2 -values. As for standard Parisi matrices [1,2], $tr\{Q_\kappa^2\} = n(m_\kappa - 1)q_{1\kappa}^2 + n(n - m_\kappa)q_{2\kappa}^2$, $\kappa = A, B$, so for a physical finite free energy one requires that $tr\{Q_3\}^2$ also scales like n for small n . A set of self-consistent equations is derived by the condition that m_A, m_B and γ extremize the free energy in the replica limit. The idea is thus to find a function of m_A, m_B and γ , which agrees with the trace of Q_{AB}^2 for integer-valued parameters m_A, m_B, n allowed by the scheme, satisfies the integral constraints ahead of equation (2), and whose analytical continuation for small n is linear in n . This goal is achieved by evaluating all overlap contributions to $tr\{Q_3^2\}$ and reexpressing the results for each of the four different overlap regions in terms of the embedding factor γ .

The trace formula for Q_3^2 for arbitrary block-sizes $m_A < m_B$, which all nest the host-matrix of size n , for the single-embedding scheme, is expressible in terms of γ as

$$\begin{aligned} \Xi(\gamma) \equiv tr\{Q_{AB}^2\} &= n(m_A - 1)(q_{1A} + q_{1B})^2 \\ &+ n(m_B - m_A)(q_{2A} + q_{1B})^2 + n(n - m_B)(q_{2A} + q_{2B})^2 \\ &+ 2(q_{1A} - q_{2A})(q_{1B} - q_{2B})\phi(\gamma), \quad (3) \end{aligned}$$

where the dependence on the embedding factor γ is contained in

$$\begin{aligned} \phi(\gamma) \equiv \frac{n}{3m_B^3} \{ &(m_B - n)(m_B - (1 + \gamma)m_A) \\ &\times [(m_B - 2n)m_B + m_A(2n(1 + \gamma) - (4 + \gamma)m_B)] \}. \quad (4) \end{aligned}$$

The result represented by equation (3), together with ϕ given by equation (4), holds for all integral and non-integral (rational) values of γ allowed by the construction. Let us consider a few examples, using this division into two classes with either integral or non-integral embedding factors γ :

i) there exists a subset of matrices, where each m_B -block hosts the same (integral) number of m_A -blocks. The left hand side of Figure 1 shows one example with $(m_A = 3, m_B = 5, n = 15, \gamma = 1)$. Further examples of this class are $(2, 7, 14, 3)$, $(4, 7, 28, 1)$, $(3, 8, 24, 2)$;

ii) the right hand side of Figure 1 presents one example for the matrix-class having non-integral embedding factors with $(m_A = 7, m_B = 11, n = 77, \gamma = 5/7)$. Let us add further examples by $(3, 4, 12, 2/3)$ and $(7, 17, 119, 11/7)$.

One can see that Ξ -contributions from all overlap regions can be expressed in terms of m_A, m_B, γ , while the number of these overlap regions depends explicitly on the host matrix size n . This feature guarantees the finite replica limit of the free energy.

3.2 Multiple-embedding modulated scheme (MES)

We also define a modulated scheme *MES* which incorporates a k -fold repeated *SES*-structure (of size $n_1 \times n_1$) along the diagonal of an $n \times n$ host matrix, for example $(4, 6, 24, 3/4)$ where $k = 2$. The *SES*-matrix size is chosen as a variational parameter, kept finite while the replica limit $n \rightarrow 0$ is taken, and finally varied to extremize F . Altogether m_A, m_B , and n_1 are variational parameters, which determine the embedding factor $\gamma(m_A, m_B, n \rightarrow n_1)$ according to equation (2), or m_A, m_B and γ are varied and their selfconsistent solutions yield $n_1(\gamma) = m_A m_B / (m_A - m_B + m_A \gamma)$. The free energy is obtained by means of $\lim_{n \rightarrow 0} \Xi(n, n_1)/n$, where

$$\Xi(n, n_1) = \Xi(\gamma(n_1))n/n_1 + n(n - n_1)(q_{2A} + q_{2B})^2 \quad (5)$$

or $\Xi(n, \gamma) = \Xi(\gamma)n/n_1(\gamma) + n(n - n_1(\gamma))(q_{2A} + q_{2B})^2$ in case γ is varied. An example for a three-fold embedding is given by Figure 2. Since relation (2) is part of the definition of the *MES* and since the replica limit maps the Parisi-type parameters m_A, m_B, n_1 from $[1, \infty]$ into the interval $[0, 1]$, γ is thus restricted to values $\gamma > -1$ in contrast to the *SES*. The *MES* also differs essentially from the *SES* by the fact that the (A,B)-symmetric limit $m_A = m_B \equiv m$ reduces it to the unmodulated 1RSB-scheme. Another possible variant of *MES*, where the number $k = n/n_1$ of repeated *SES*-structures is varied, is discarded, since k cannot extremize F .

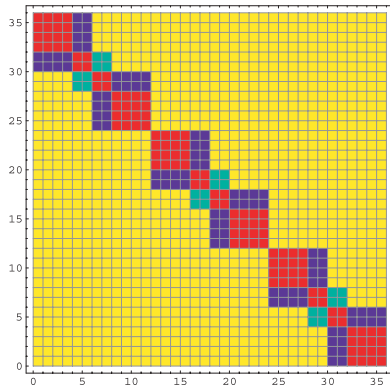


Fig. 2. Example of a modulated saddle point matrix belonging to the multiple-embedding scheme: $\langle Q_3 \rangle/i = \langle Q_A \rangle + \langle Q_B \rangle$ with $m_A = 4$, $m_B = 6$, and $n = 36$ in one-step *RSB*.

3.3 Upgrade of the modulated RSB-schemes

While the modulated *RSB*-schemes are initially designed for applications to glassy ferrimagnetic systems, one should also consider them under a more general point of view: they can be used in systems without sub-lattices and even without reference to antiferro- or ferrimagnetic order. One can upgrade the schemes as an alternative of the Parisi-scheme provided the traces of higher powers Q^k , $k \geq 3$, also yield a non-divergent replica-limit. This separate point as well as higher order *RSB* is not the issue of the present article, where only trQ_3^2 is needed for the *KS*-model.

4 Application to a layer model with a confined magnetic field

In order to examine the specific features of the modulated *SES*-scheme in comparison with the unmodulated one, we performed a detailed analysis for the *KS*-model (1) in a spatially confined magnetic field H_p . For an equal number of αN of *A*- and *B*-spins we chose $h = H_p$ and $h = 0$ for the remaining $(1 - \alpha)N$ spins.

The free energy of this model realization in 1-step *RSB* can be decomposed into three parts

$$F = F_0 + \alpha F_1(H_p) + (1 - \alpha) F_1(H_p = 0), \quad (6)$$

where

$$F_0 = -J_{af} M_A M_B - \frac{J^2}{4T} \left[\lim_{n \rightarrow 0} \frac{1}{n} tr \underline{Q}^2 - 2 \sum_{\kappa} (1 - q_{1\kappa}) \right], \quad (7)$$

and

$$F_1(H_p) = - \sum_{\kappa=A,B} \frac{T}{m_{\kappa}} \int_{z_{2\kappa}}^G \ln \int_{z_{1\kappa}}^G \cosh^{m_{\kappa}} \left\{ \frac{1}{T} \tilde{H}_{\kappa}(H_p) \right\} \quad (8)$$

with $\int_z^G \equiv \int_{-\infty}^{\infty} dz e^{-z^2/2} / \sqrt{2\pi}$, and $\underline{Q} \equiv (Q_A, Q_B, Q_3)$.

The effective field \tilde{H}_{κ} on sublattice κ depends on the order

parameters of the complementary sublattice $\bar{\kappa}$. It is given in terms of 1. the confined polaron field H_{κ} , 2. in terms of the magnetization field of the complementary sublattice $\bar{\kappa}$, and 3. spin fields representing the spin glass field $z_{k,\kappa}$ (in the *1RSB*-discretized approximation), by the expression

$$\tilde{H}_{\kappa}(H_p) = H_p - J_{af} M_{\bar{\kappa}} + J \sqrt{q_{2\bar{\kappa}}} z_{2\kappa} + J \sqrt{q_{1\bar{\kappa}} - q_{2\bar{\kappa}}} z_{1\kappa}, \quad (9)$$

The motivation for choosing this specification of the *KS*-model is essentially twofold:

1) by scanning the full range $0 < \alpha < 1$ we found that the phase diagram is not only marked by a continuous spin glass–ferrimagnet transition. At slightly higher ratios J_{af}/J , a subsequent small flop transition from ferrimagnet to an (what might be called) antiferrimagnetic phase occurs, which provides an ideal test-ground for the *SES*-scheme:

the $A \leftrightarrow B$ -symmetries are strongly broken, since solutions are far away from either $M_A = M_B$ or $M_A = -M_B$. The main features of the phase diagram for all α at selected characteristic polaron fields H_p and temperatures is analyzed below in 5.

2) CdTe/Cd_{1-x}Mn_xTe-layers are well described by the present model, where H_p represents a confined polaron field created by polarized exciton-hole spins being localized at the interface. The penetration depth of the hole-wavefunction defines the portion α of the CdMnTe-layer which is exposed to the field H_p . The magnetic CdMnTe-layer employs Villain-Ising pseudo-spins S [16] representing tetrahedra of manganese Heisenberg-spins which retain only two orientational degrees of freedom. All pseudospins of the magnetic layer are then coupled by a long-range partially frustrated interaction with antiferromagnetic mean value. This model (with $\alpha = 0.5$) provided optimal fits for experiments in the spin glass regime at $x = 33\%$ [14]. Increasing Mn-concentration x enhances the antiferromagnetic bias and eventually leads to a transition from spin glass to antiferromagnetic or ferrimagnetic order (in a homogeneous polaron- or external field) at a critical concentration x_c . Scaling and numerical analysis on the basis of anisotropic Heisenberg models were also provided [17, 18]. The virtue of the Pseudo-Ising concept lies in the smaller lower critical dimension when compared to Heisenberg systems.

We explore at $T = 0$ the difference between the *SES*-scheme and the unmodulated one. For its demonstration we choose a polaron-field strength $H_p = 4J$ and a confinement-fraction $\alpha = 0.5$ (half-penetrated layer). Our 1-step *RSB*-results are obtained by solving up to seven coupled selfconsistent integral equations (*SES*-scheme) which extremize F . Thanks to the $T \rightarrow 0$ -limit one integration can be solved exactly, simplifying the selfconsistent set of ten coupled double-integral equations for finite T considerably (while $q_{1A} = q_{1B} = 1$ at $T = 0$, both $q_{1\kappa}(T)$, $\kappa = A, B$, and the hole polarization too must be determined selfconsistently for finite T). The $T = 0$ -results of Figures 3, 4 show a continuous SG-ferrimagnetic transition with order parameter $M_A - M_B$ to occur at $r \equiv J_{af}/J \approx 3.25$, followed by a discontinuous transition to antiferrimagnetic order near $r \approx 4.02$. As Figure 3 shows, q_{2A} and q_{2B}

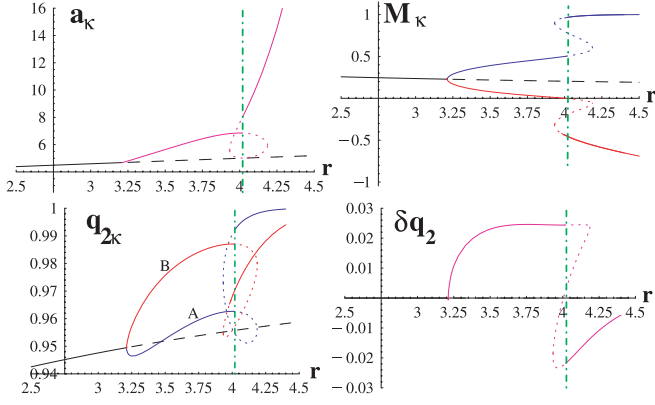


Fig. 3. Solutions for $T = 0$ and $\alpha = 0.5$ in 1-step unmodulated *RSB*: Parisi parameter $a_A = a_B$, $a_\kappa = m_\kappa/T$, sublattice magnetizations M_A, M_B , spin glass order parameters q_{2A}, q_{2B} , and $\delta q_2 \equiv q_{2B} - q_{2A}$, as a function of $r \equiv J_{af}/J$ and finite confined field $H_p = 4J$. Dash-dotted lines locate the 1st-order transition, dashed lines show unstable solutions.

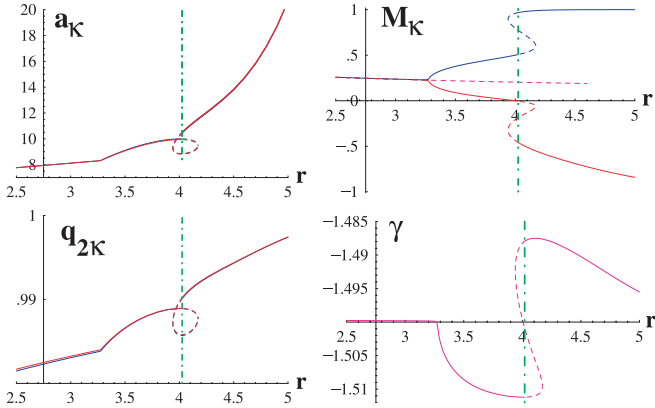


Fig. 4. Corresponding results in the modulated *RSB*-scheme *SES* showing (a_A, a_B) , (M_A, M_B) , (q_{2A}, q_{2B}) , and the *SES*-embedding parameter γ . It is seen that a_κ and q_κ show only very weak sublattice splitting.

undergo large jumps and become (almost) interchanged at the discontinuous transition in the framework of the standard unmodulated scheme, i.e. under the condition $a_A = a_B$, $a_\kappa \equiv \lim_{T \rightarrow 0} m_\kappa/T$.

In the modulated *SES*-scheme, Figure 4, the selfconsistent solutions for a_A, a_B and q_{2A}, q_{2B} are different but show only small sublattice splitting; at the transition $a_B - a_A$ changes sign, in contrast to $q_B - q_A$.

The sublattice effective fields of equation (6) help to explain the origin of the discontinuous transition: the competition between $J_{af}M_{\bar{r}}$ and the polaron field H_p leads (in the clean limit) eventually to a total spin reversal on one sublattice. Random magnetic order would smear the jump in any homogeneous field but the discontinuity reappears due to the competition between antiferromagnetic ($\{A, B\}$ -symmetric) order, preferred in the $H_p = 0$ -region, and a strongly (A, B) -asymmetric ferrimagnetic order for sufficiently large H_p/J .

We finally compare our *1RSB*-results for the free energies in the double transition regime of Figures 3, 4. Fig-

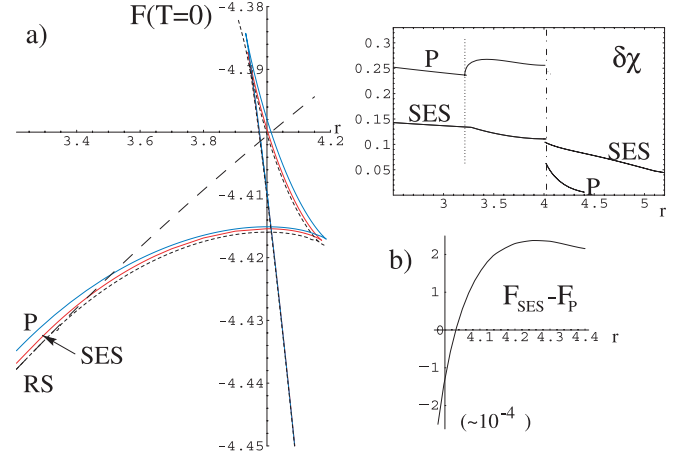


Fig. 5. a) Energies $F(T = 0)$ of modulated *1RSB*- (arrow), unmodulated *1RSB*- (*P*), and *RS*-scheme (*RS*) with ferromagnetic *RS*-solution (dashed), shown as a function of $r \equiv J_{af}/J$ in the double transition region, b) energy difference between modulated- and unmodulated scheme; inset (top right) shows susceptibility *RSB*-contributions $\delta\chi$.

ure 5 provides evidence for the discontinuous transition to involve a transition from unmodulated to modulated *RSB*. As discussed in [1] higher energies correspond to improved solutions (unless identical stability criteria are met). Figure 5b shows that the energy for the *SES*-scheme is higher for $J_{af} > J_{af}^c \approx 4.02$ (lower if $<$ holds). This crossing of energies at the discontinuous transition and the character of the *RSB*-schemes suggests that modulated *RSB* governs the $J_{af} > J_{af}^c$ -regime. The *RS*-solution is lowest but unstable everywhere. Despite small energy separation, equilibrium- and nonequilibrium linear susceptibility shows large differences which depend strongly on the type of *RSB*-scheme (inset of Fig. 5). Further observable manifestations and thermal behavior remain to be considered.

5 Confined field-fraction effect on the multiplicity of phase transitions

The sublattice effective fields of equation (9) help to explain the origin of the discontinuous transition: the competition between $J_{af}M_{\bar{r}}$ and the polaron field leads (in the clean limit) to a total spin reversal on one sublattice. Random magnetic order would smear the jump in any homogeneous field, as can be deduced from Figure 6 at $\alpha = 1$, but the discontinuity reappears due to the competition between antiferromagnetic ((A, B) -symmetric) order, preferred in the $H_p = 0$ -region, and a strongly (A, B) -asymmetric ferrimagnetic order for sufficiently large $H_p \neq 0$. For half-penetrated layer ($\alpha = 0.5$) the continuous transition with order parameter $M_A - M_B$, caused by the competition between spin glass and antiferromagnetic order, exists at $J_{af} = J_{af}^c \approx 3.25J$. To understand the $\alpha = 0.5$ -scenario in the context of all $0 \leq \alpha \leq 1$ we found sufficient to analyze the stability limits at $T = 0$

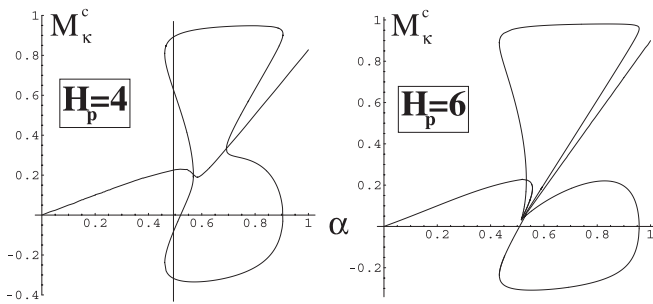


Fig. 6. Magnetizations $M_{\kappa=A,B}^c$ obeying the constraint $dM_{\kappa}^c/dr = \infty$, $r \equiv J_{af}/J$, are shown in RS -approximation for $T = 0$ as a function of the polaron fraction α for field strengths $H_p = 4, 6$; the vertical line indicates the position of the double transition analyzed in $1RSB$ -schemes in Figures 3, 4.

in an RS -approximation. Introducing the definitions

$$\xi = r M_{\kappa}, h_p \equiv H_p/J \quad (10)$$

one may cast the equation of state into the compact nested form

$$\xi = u(u(\xi) + \eta u(\xi - h_p)) + \eta u(u(\xi) + \eta u(\xi - h_p) + h_p), \quad (11)$$

where

$$u(\xi) \equiv (1 - \alpha) r \operatorname{erf}(\xi/\sqrt{2}), \eta = \alpha/(1 - \alpha). \quad (12)$$

The stability limits are obtained under the constraint

$$\frac{dr(M_{\kappa})}{dM_{\kappa}} = 0, \frac{d^2r(M_{\kappa})}{dM_{\kappa}^2} \neq 0 \quad (13)$$

in terms of the inverted solution $r(M_{\kappa})$ displaying the interaction ratio $r \equiv J_{af}/J$ as a function of the sublattice magnetizations M_{κ} . The constrained solutions M_{κ}^c are derived by scanning all α and shown in Figure 6 for typical values of H_p (stability limits in terms of $J_{af}(\alpha)$ are omitted for brevity). At a fixed α , the existence of one or three solutions implies a single continuous or a single discontinuous SG-ferrimagnetic transition respectively, while five solutions are necessary to obtain a double transition regime (with a magnetization-curve $M(r = J_{af}/J)$ shaped as in Figs. 3, 4). Corrections in the effective field \tilde{H} originating from intra-sublattice interactions do not change qualitatively the results. The right part of Figure 6 shows a special point which emerges for large H_p near $\alpha \approx 0.5$ and small M_{κ}^c .

The Korenblit-Shender model in a field $h = H_p$ can be retrieved at $\alpha = 1$.

6 Conclusions and open ends

In this article hybrid RSB -schemes which combine tree- and wave-like structures were defined. A variational embedding factor of Parisi block matrices appeared as a characteristic ingredient of the new schemes.

In the evaluated leading order of one-step RSB the application showed that the SES -modulated phase appears to be preferred beyond a critical coupling J_{af}^c where a type of glassy (anti-)ferrimagnetic order prevails.

The MES -scheme (in contrast to SES) allows a continuous crossover to unmodulated RSB . Like the SES , it should also be extended to infinite order of symmetry breaking and analyzed for its stability, both near T_c and at $T = 0$.

A generalization of our trace-formula to all powers of the order parameter matrix can in principle create an extension of the Parisi scheme, modelling perhaps excited states in short-range spin glasses. Viewing replica symmetry breaking as a fluctuational effect itself, it seems reasonable to incorporate spatial fluctuations of even more general type than staggered order in the Parisi scheme. Higher orders of both types of RSB are currently under study for ferrimagnets, using for example techniques applied to unmodulated RSB in reference [19].

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